Progress in the Gauge Theory with Spontaneously Broken $\mathcal{N}=2$ Supersymmetry

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paper to appear

cf. H. I., K. Maruyoshi and S. Minato
arXiv:0909.5486, Nucl. Phys. B **830**

K. Fujiwara and, H.I. and M. Sakaguchi
arXiv: hep-th/0409060, P. T. P. **113**
arXiv: hep-th/0503113, N. P. B **723**

Plan of my talk:

I) $\mathcal{N}=2$ action with tree vacuum $\mathcal{N}=2 \rightarrow \mathcal{N}=1$
nonabelian gauge group, A.P.T. for U(1), FIS1

II) Several properties: mass spectrum, low energy theorem for NGF, FIS3, IMM

III) Dynamically realizing $\mathcal{N}=1 \rightarrow \mathcal{N}=0$ by $\langle D^0 \rangle \neq 0$, gap eq.
I-Maru

I) Action to work with

$$\mathcal{L} = \mathrm{Im}\left[\int d^4\theta \mathrm{Tr}\bar{\Phi} e^{adV}\frac{\partial \mathcal{F}(\Phi)}{\partial \Phi} + \int d^2\theta \frac{1}{2}\frac{\partial^2\mathcal{F}(\Phi)}{\partial\Phi^a\partial\Phi^b}\mathcal{W}^{\alpha a}\mathcal{W}^b_\alpha\right] + \left(\int d^2\theta W(\Phi) + c.c.\right)$$

$$W(\Phi) = \mathrm{Tr}\left(2e\Phi + m\frac{\partial\mathcal{F}(\Phi)}{\partial\Phi}\right)$$

- U(N) gauge group assumed for definiteness (product gauge group O.K.)
- $\mathcal{F}(\Phi)$: prepotential, input function
- superpotential W supplied by the electric and magnetic FI terms, made possible by a particular fixing of rigid $SU(2)_R$ symmetry
- should contrast with $\mathcal{L}_W^{\mathcal{N}=1} = \int d^4\theta \mathrm{Tr}\bar{\Phi}e^{adV}\Phi + \left[\int d^2\theta \mathrm{Tr}\left(i\tau\mathcal{W}\mathcal{W} + W(\Phi)\right) + h.c.\right]$
- In III), will work with

$$S[\mathcal{F}] = \int d^4x \mathcal{L}$$

$$S_{\mathrm{bare}} = S[\mathcal{F}] + S_{\mathrm{c.t.}} \qquad\qquad S_{\mathrm{c.t.}} = S\left[\mathcal{F} = \Lambda(\Phi^0)^2/2\right]$$

limiting cases & description at lower energy

$\mathcal{N}=1$ large | interpolation by e(complex), m | small $\mathcal{N}=2$

μ_0 $\qquad S_W^{\mathcal{N}=1} \qquad S[\mathcal{F}] \qquad S_{\mathrm{SYM}}^{\mathcal{N}=2}$

μ $\qquad F_{\text{free}}(S_i), W_{\text{eff}}$

$$W_{\text{eff}} = \sum_i N_i \frac{\partial F_{\text{free}}}{\partial S_i}$$

DV

R.S. & matrix model description

$\cdots$

μ_{IR}

$$W_{\text{eff}} = \sum_{i=1}^{n} N_i \frac{\partial F_{\text{free}}}{\partial S_i} + \frac{16\pi^2 i}{m} \sum_{k=2}^{n+1} g_k \frac{\partial F_{\text{free}}}{\partial g_{k-1}}$$

$\mathcal{F}_{\text{SW}}^{(\text{eff})}(\phi_i)$ SW

R.S. description

H.I. -Maruyoshi
arXiv:0704.1060, P. L. B **650**
arXiv:0710.4377, N. P. B **796**

$\mathcal{N}=2$ susy of $\mathcal{L}$ and tree vacua

- construction of 2nd susy δ_{η_2} : Let $\boldsymbol{R}$ be

$$\begin{pmatrix} \lambda^a \\ \psi^a \end{pmatrix} \to \begin{pmatrix} \psi^a \\ -\lambda^a \end{pmatrix}$$

$R\delta^{(1,\mathrm{Im}e)}_{\eta_1=\theta}R^{-1} \equiv \delta^{(2,-\mathrm{Im}e)}_{\eta_2=\theta}$ so that $0=\delta^{(2,\mathrm{Im}e)}_{\eta_2=\theta}S(\mathrm{Im}e)$ follows from $R\delta^{(1,\mathrm{Im}e)}_{\eta_1=\theta}S(\mathrm{Im}e)R^{-1}=0$

- the form of W and $\tau_{ab}=\mathcal{F}_{ab}$ are derived by imposing $\mathcal{R}$
- $V_{\mathrm{tree}} = V^{(D)} + V^{(\mathrm{sup})}, \quad V^{(\mathrm{sup})} = g^{ab}\partial_a W \overline{\partial_b W}$

  where $V^{(D)} = -\frac{1}{2}g_{ab}D^a D^b + \cdots$

- $e\delta^0_c + m\mathcal{F}_{0c} = 0$; vacuum condition
- $\langle \delta_{\eta_2}\lambda^a\rangle = -\sqrt{2}\eta_2\langle \tilde{F}^a\rangle \propto \delta^0_a(\mathrm{Im}e)$ $\quad\therefore$ 2nd susy broken
- generic breaking pattern of gauge symmetry: $\deg \mathcal{F} = n+2$

$$U(N) \to \prod_{i=1}^{n} U(N_i) \quad \text{with} \quad \sum_{i=1}^{n} N_i = N$$

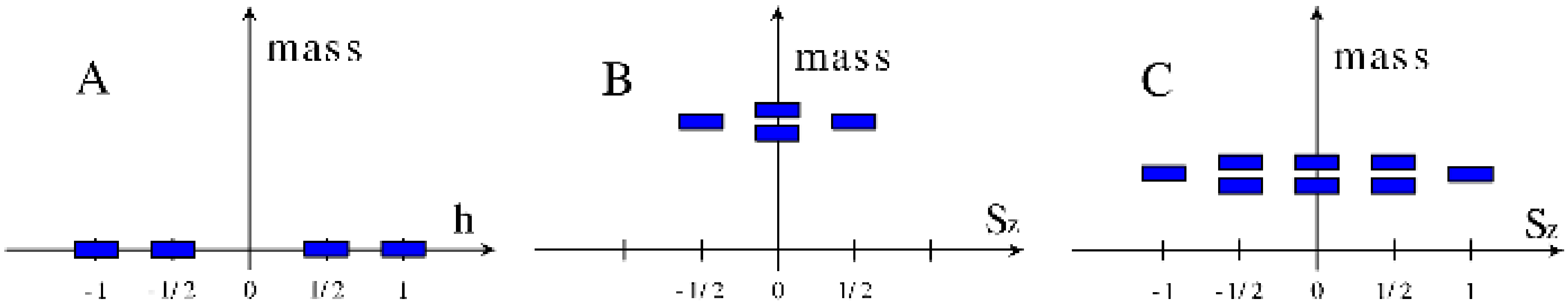
II) mass spectrum (tree level)

index labelling $a, b, \cdots = \begin{cases} \alpha, \beta, \cdots \text{ for unbroken generators} \\ \mu, \nu, \cdots \text{ for broken generators} \end{cases}$

- the table

field	mass	label	# of polarization states
v_m^{α}	0	A	$2d_u (d_u \equiv \dim \prod_i U(N_i))$
v_m^{μ}	$\lvert \frac{1}{\sqrt{2}} f^{\nu}_{\mu \underline{i}} \lambda^{\underline{i}} \rvert$	C	$3(N^2 - d_u)$
λ^{α}	0	A	$2d_u$
ψ^{α}	$\lvert m \langle\langle g^{\alpha\alpha} \rangle\rangle \langle\langle \mathcal{F}_{0\alpha\alpha} \rangle\rangle \rvert$	B	$2d_u$
$\boldsymbol{\lambda}_I^{\mu}$	$\lvert \frac{1}{\sqrt{2}} f^{\nu}_{\mu \underline{i}} \lambda^{\underline{i}} \rvert$	C	$4(N^2 - d_u)$
$\tilde{\phi}^{\alpha}$	$\lvert m \langle\langle g^{\alpha\alpha} \rangle\rangle \langle\langle \mathcal{F}_{0\alpha\alpha} \rangle\rangle \rvert$	B	$2d_u$
$\mathcal{P}_{\mu}^{\tilde{\mu}} \tilde{\phi}^{\mu}$	$\lvert \frac{1}{\sqrt{2}} f^{\nu}_{\mu \underline{i}} \lambda^{\underline{i}} \rvert$	C	$N^2 - d_u$

- $\mathcal{N} = 1$ supermultiplet



low energy suppression of processes with NGF emission

- low energy theorem on $S^{(2)\mu}$, namely,

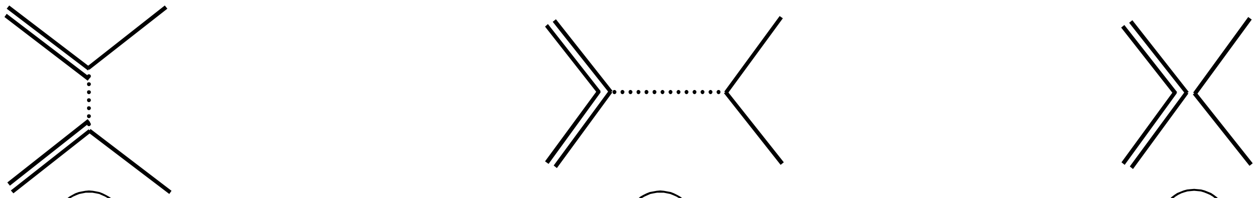
$$\text{F.T.}\langle p_f;\cdots|\mathcal{S}^{(2)\mu}|p_i;\cdots\rangle(q) = q^\mu F(q^2,\cdots) + R^\mu(q^2,\cdots) \quad \text{valid}$$

$$\Rightarrow \quad \lim_{q^\mu\to 0} q^2 F(q^2,\cdots) = 0$$

- on the other hand, $\exists$ NGF-SU(N) fermions nonderivative couplings responsible for ψ^α mass

- the resolution is the cancellation:

consider $\psi^a\psi^b \to \lambda^0\lambda^\alpha$ scattering $((a,b)=(\alpha,0))$



1 2 3

1 ; t-channel diagram, $f_{0ab}=0$ $\therefore$ vanishes

2 + 3 $\xrightarrow[p_0^\mu \to 0]{}$ 0, saving the theorem

III) Self-consistent Hartree-Fock approximation

- For simplicity, consider the case U(N) unbroken
- hunt for the possibility (up to one-loop):

$\langle D^0 \rangle \neq 0$ $\Rightarrow$ Mixed Maj.-Dirac mass to $\mathcal{N}=2$ gaugino,

$\mathcal{N}=1 \rightarrow \mathcal{N}=0$

- We noted

$$\mathcal{L}_{\text{gauge}} \ni -\frac{1}{2}(\lambda^a, \psi^a) \begin{pmatrix} 0 & -\frac{\sqrt{2}}{4}\mathcal{F}_{abc}D^b \\ -\frac{\sqrt{2}}{4}\mathcal{F}_{abc}D^b & \partial_a\partial_c W \end{pmatrix} \begin{pmatrix} \lambda^c \\ \psi^c \end{pmatrix} + (c.c.)$$

no such coupling to bosons present

$$\langle D^0 \rangle = -\frac{1}{2\sqrt{2}} \langle g^{0b}\mathcal{F}_{bcd}\psi^d\lambda^c + g^{0b}\bar{\mathcal{F}}_{bcd}\bar{\psi}^d\bar{\lambda}^c \rangle \qquad \therefore \text{ DSB}$$

- **$V_{\text{1-loop}}$:**

mass matrix (holomorphic and nonvanishing part)

$$M_{Fa} \equiv \begin{pmatrix} 0 & -\frac{\sqrt{2}}{4}\langle \mathcal{F}_{0aa} D^0 \rangle \\ -\frac{\sqrt{2}}{4}\langle \mathcal{F}_{0aa} D^0 \rangle & \langle \partial_a \partial_a W \rangle \end{pmatrix}$$

The eigenvalues are

$$m_a \lambda^{(\pm)}, \qquad \lambda^{(\pm)} \equiv \frac{1}{2}\left(1 \pm \sqrt{1+\Delta^2}\right), \quad \Delta^2 \equiv \frac{(D^0)^2}{4Nm^2}$$

We obtain

$$\frac{1}{\sum | m_a |^4} V_{1-\text{loop}} = \frac{1}{32\pi^2}\left(A(d)(\Delta^2 + \frac{1}{8}\Delta^4) - \lambda^{(+)4}\log\lambda^{(+)2} - \lambda^{(-)4}\log\lambda^{(-)2}\right)$$

where

$$A(d) = \frac{3}{4} - \gamma + \frac{1}{2-d/2}$$

- $V^{(D)}_{1-\mathrm{loop}} = V^{(D)} + V_{\mathrm{c.t.}} + V_{1-\mathrm{loop}}$ **:**

In order to trade A with Λ in $V_{\mathrm{c.t.}}$,

impose, for instance,

$$\frac{1/2}{\sum \mid m_a \mid^4} \frac{\partial^2 V}{(\partial \Delta)^2}\bigg|_{\Delta=0} = c \quad \text{(some number),}$$

we obtain

$$\frac{1}{\sum \mid m_a \mid^4} V^{(D)}_{1-\mathrm{loop}} = \left(c + \frac{1}{64\pi^2} \right) \Delta^2 + \Lambda'_{\mathrm{res}} \frac{\Delta^4}{8} - \frac{1}{32\pi^2} \left(\lambda^{(+)4} \log \lambda^{(+)2} + \lambda^{(-)4} \log \lambda^{(-)2} \right)$$

- **gap equation:**

is a stationary condition to $V^{(D)}_{\text{1-loop}}$ $\Rightarrow$ a solution $\Delta = \Delta^* \neq 0$ exists

$\mathcal{N} = 1$ susy is broken to $\mathcal{N} = 0$.

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